



MINISTRY OF EDUCATION, SINGAPORE
in collaboration with
CAMBRIDGE INTERNATIONAL EDUCATION
General Certificate of Education Advanced Level

List MF27

LIST OF FORMULAE

AND

RESULTS

for Mathematics and Further Mathematics

For use from 2025 in all papers for the H1, H2 and H3 Mathematics and
H2 Further Mathematics syllabuses.

This document has 8 pages.



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Algebraic series

Binomial expansion:

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \binom{n}{3} a^{n-3}b^3 + \dots + b^n, \text{ where } n \text{ is a positive integer}$$

$$\text{and } \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Maclaurin expansion:

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \dots + \frac{x^n}{n!}f^{(n)}(0) + \dots$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^r}{r!} + \dots \quad (\text{all } x)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + \frac{(-1)^r x^{2r+1}}{(2r+1)!} + \dots \quad (\text{all } x)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + \frac{(-1)^r x^{2r}}{(2r)!} + \dots \quad (\text{all } x)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + \frac{(-1)^{r+1} x^r}{r} + \dots \quad (-1 < x \leq 1)$$

Partial fractions decomposition

Non-repeated linear factors:

$$\frac{px+q}{(ax+b)(cx+d)} = \frac{A}{(ax+b)} + \frac{B}{(cx+d)}$$

Repeated linear factors:

$$\frac{px^2+qx+r}{(ax+b)(cx+d)^2} = \frac{A}{(ax+b)} + \frac{B}{(cx+d)} + \frac{C}{(cx+d)^2}$$

Non-repeated quadratic factor:

$$\frac{px^2+qx+r}{(ax+b)(x^2+c^2)} = \frac{A}{(ax+b)} + \frac{Bx+C}{(x^2+c^2)}$$

Trigonometry

$$\sin(A \pm B) \equiv \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) \equiv \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) \equiv \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A \equiv 2 \sin A \cos A$$

$$\cos 2A \equiv \cos^2 A - \sin^2 A \equiv 2 \cos^2 A - 1 \equiv 1 - 2 \sin^2 A$$

$$\tan 2A \equiv \frac{2 \tan A}{1 - \tan^2 A}$$

Principal values:

$$-\frac{1}{2}\pi \leq \sin^{-1}x \leq \frac{1}{2}\pi \quad (|x| \leq 1)$$

$$0 \leq \cos^{-1}x \leq \pi \quad (|x| \leq 1)$$

$$-\frac{1}{2}\pi < \tan^{-1}x < \frac{1}{2}\pi$$

Derivatives

$f(x)$	$f'(x)$
$\sin^{-1}x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1}x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1}x$	$\frac{1}{1+x^2}$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sec x$	$\sec x \tan x$

Integrals

(Arbitrary constants are omitted; a denotes a positive constant.)

$f(x)$	$\int f(x) dx$	
$\frac{1}{x^2 + a^2}$	$\frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$	
$\frac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)$	$(x < a)$
$\frac{1}{x^2 - a^2}$	$\frac{1}{2a} \ln\left(\frac{x-a}{x+a}\right)$	$(x > a)$
$\frac{1}{a^2 - x^2}$	$\frac{1}{2a} \ln\left(\frac{a+x}{a-x}\right)$	$(x < a)$
$\tan x$	$\ln(\sec x)$	$(x < \frac{1}{2}\pi)$
$\cot x$	$\ln(\sin x)$	$(0 < x < \pi)$
$\operatorname{cosec} x$	$-\ln(\operatorname{cosec} x + \cot x)$	$(0 < x < \pi)$
$\sec x$	$\ln(\sec x + \tan x)$	$(x < \frac{1}{2}\pi)$

Vectors

The point dividing AB in the ratio $\lambda : \mu$ has position vector $\frac{\mu \mathbf{a} + \lambda \mathbf{b}}{\lambda + \mu}$

Vector product:

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$$

Applications of definite integrals

Arc length of a curve defined in cartesian form: $\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$

Surface area of revolution about the x -axis for a curve defined in cartesian form:

$$\int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

Functions of two variables

Quadratic approximation of f at (a, b) :

$$f(x, y) \approx f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b) + \frac{1}{2} f_{xx}(a, b)(x - a)^2 + f_{xy}(a, b)(x - a)(y - b) + \frac{1}{2} f_{yy}(a, b)(y - b)^2$$

Numerical methods

Trapezium rule (for single strip): $\int_a^b f(x)dx \approx \frac{1}{2}(b-a)[f(a)+f(b)]$

Simpson's rule (for two strips): $\int_a^b f(x)dx \approx \frac{1}{6}(b-a) \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$

The Newton-Raphson iteration for approximating a root of $f(x) = 0$:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)},$$

where x_1 is a first approximation.

Euler Method with step size h :

$$y_2 = y_1 + hf(x_1, y_1)$$

Improved Euler Method with step size h :

$$u_2 = y_1 + hf(x_1, y_1)$$

$$y_2 = y_1 + \frac{h}{2} [f(x_1, y_1) + f(x_2, u_2)]$$

Standard discrete distributions

Distribution of X	$P(X = x)$	Mean	Variance
Binomial $B(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$
Poisson $Po(\lambda)$	$e^{-\lambda} \frac{\lambda^x}{x!}$	λ	λ
Geometric $Geo(p)$	$(1-p)^{x-1} p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$

Standard continuous distribution

Distribution of X	p.d.f.	Mean	Variance
Exponential	$\lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$

Sampling and testing

Unbiased estimate of population variance:

$$s^2 = \frac{n}{n-1} \left(\frac{\Sigma(x-\bar{x})^2}{n} \right) = \frac{1}{n-1} \left(\Sigma x^2 - \frac{(\Sigma x)^2}{n} \right)$$

Regression and correlation

Estimated product moment correlation coefficient:

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\{\Sigma(x-\bar{x})^2\} \{\Sigma(y-\bar{y})^2\}}} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\sqrt{\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n} \right) \left(\Sigma y^2 - \frac{(\Sigma y)^2}{n} \right)}}$$

Estimated regression line of y on x :

$$y - \bar{y} = b(x - \bar{x}), \quad \text{where } b = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$$

WILCOXON SIGNED RANK TEST

P is the sum of the ranks corresponding to the positive differences,
 Q is the sum of the ranks corresponding to the negative differences,
 T is the smaller of P and Q .

For each value of n the table gives the **largest** value of T which will lead to rejection of the null hypothesis at the level of significance indicated.

Critical values of T

	Level of significance			
One	0.05	0.025	0.01	0.005
Two	0.1	0.05	0.02	0.01
$n = 6$	2	0		
7	3	2	0	
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15
16	35	29	23	19
17	41	34	27	23
18	47	40	32	27
19	53	46	37	32
20	60	52	43	37

Mathematical Results

AM-GM inequality:

For any nonnegative real numbers x_1, x_2, \dots, x_n ,

$$\frac{x_1 + x_2 + \dots + x_n}{n} \geq \sqrt[n]{x_1 x_2 \dots x_n},$$

where the equality holds if and only if $x_1 = x_2 = \dots = x_n$.

Cauchy-Schwarz inequality:

For any real numbers u_1, u_2, \dots, u_n and v_1, v_2, \dots, v_n ,

$$\left(\sum_{i=1}^n u_i v_i \right)^2 \leq \left(\sum_{i=1}^n u_i^2 \right) \left(\sum_{i=1}^n v_i^2 \right),$$

where the equality holds if and only if there exists a nonzero constant k such that $u_i = kv_i$ for all $i = 1, 2, \dots, n$.

Triangle inequality:

For any real numbers x_1, x_2, \dots, x_n ,

$$|x_1 + x_2 + \dots + x_n| \leq |x_1| + |x_2| + \dots + |x_n|,$$

where the equality holds if x_1, x_2, \dots, x_n are all nonnegative.

Inclusion-Exclusion Principle:

For any subsets A_1, A_2, \dots, A_n of a set,

$$\begin{aligned} |A_1 \cup A_2 \cup \dots \cup A_n| &= |A_1| + |A_2| + \dots + |A_n| \\ &\quad - [|A_1 \cap A_2| + |A_1 \cap A_3| + \dots + |A_{n-1} \cap A_n|] \\ &\quad + [|A_1 \cap A_2 \cap A_3| + |A_1 \cap A_2 \cap A_4| + \dots + |A_{n-2} \cap A_{n-1} \cap A_n|] \\ &\quad \vdots \\ &\quad + (-1)^{n-1} |A_1 \cap A_2 \dots \cap A_{n-1} \cap A_n| \end{aligned}$$

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